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Tests of an Indifference Rule in
Variable-Delay and Double-Reward
Choice Procedures with Humans

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Abstract

Four-hundred and fifty participants were recruited from Amazon Mechanical Turk across three experiments to test the predictions of a hyperbolic discounting equation in accounting for human choices involving variable delays or multiple rewards (Mazur, 1984, 1986). In Experiment 1, participants made hypothetical choices between two monetary alternatives, one consisting of a fixed delay and another consisting of two delays of equal probability (i.e., a variable-delay procedure). In Experiment 2, participants made hypothetical monetary choices between a single, immediate reward and two rewards, one immediate and one delayed (i.e., a double-reward procedure). Experiment 3 also used a double-reward procedure, but with two delayed rewards. Participants in all three experiments also completed a standard delay-discounting task. Finally, three reward amounts were tested in each type of task ($100, $1000, and $5000). In the double-reward conditions (Experiments 2 and 3), the results were in good qualitative and quantitative agreement with Mazur’s model (1984, 1986). In contrast, when participants made choices involving variable delays (Experiment 1), there was relatively poor qualitative and quantitative agreement with this model. These results, along with our previous findings, suggest the structure of questions in hypothetical tasks with humans can be a strong determinant of the choice pattern.
Tests of an Indifference Rule in Variable-Delay and Double-Reward Choice Procedures with Humans

It is well known that when animals must choose between an immediate and a delayed reinforcer, the value or effectiveness of the delayed reinforcer decreases with increasing delay. To describe how reinforcer value declines as a function of delay, Mazur (1987) proposed the following hyperbolic equation:

\[ V = \frac{A}{1 + KD} \]  

(1)

where \( V \) is the value of a reinforcer delivered after a delay of \( D \) seconds, \( A \) is the amount of reinforcement, and \( K \) is a parameter that determines the rate of discounting (with higher values of \( K \) representing faster discounting as delay increases). By testing a variety of different delay-amount combinations, Mazur found that Equation 1 provided a good description of pigeons’ choices. Other studies with rats, pigeons, and nonhuman primates have also obtained results consistent with this hyperbolic equation (e.g., Richards, Mitchell, de Witt, & Seiden, 1997; Rodriguez & Logue, 1988; Woolverton, Myerson, & Green, 2007).

Research with human participants has also examined how the value of a reward decreases with delay. In many studies, people are asked to make choices between hypothetical money rewards (e.g., $500 now versus $1000 in one year). By asking a series of questions with different amounts of an immediate reinforcer, researchers can estimate indifference points (two amount-delay combinations that a person finds equally preferable). For example, one person’s answers might reveal that $750 delivered immediately is about equally preferred to $1000 delivered after one year. Studies of this type have used hypothetical questions with delays ranging from 1 day to 25 years (and occasionally longer), obtaining an indifference point at each delay. Many studies
with humans have found that the same hyperbolic equation accurately describes how reward value decreases with increasing delay (e.g., Bickel, Odum, & Madden, 1999; Johnson, Herrmann, & Johnson, 2015; Kaplan, Reed, & McKerchar, 2014; Reynolds & Schiffbauer, 2004).

Another recurrent finding in research with nonhumans is that they show a preference for variable over fixed delays to reinforcement (e.g., Cicerone, 1976; Mazur, 1984; Rider, 1983). For example, if one alternative always delivers a reinforcer after a delay of 10 s, and a second alternative delivers a reinforcer after a delay that might be either 2 or 18 s (with equal probability), animals will show a strong preference for the variable alternative, even though the average delay to reinforcement is 10 s for both alternatives. To account for this preference for variability, Mazur (1984) used a generalized form of Equation 1, as follows:

\[
V = \sum_{i=1}^{n} P_i \left( \frac{A}{1 + KD_i} \right),
\]

where the variable alternative includes \(n\) possible delays, and \(P_i\) is the probability that a delay of \(D_i\) seconds will occur. In essence, this equation states that the value of an alternative with variable delays is a weighted average of the values of all the possible delays that might occur. To show how Equations 1 and 2 predict a preference for variability, we can arbitrarily set both \(K\) and \(A\) equal to 1. Solving Equation 1 for a fixed delay of 10 s yields a value of \(V = 1/11 = 0.09\). Solving Equation 2 for a variable delay of 2 or 18 s (each with a probability of .5) yields a value of \(V = .5(1/3) + .5(1/19) = 0.19\). Because \(V\) is larger for the variable delay, these equations predict that an animal will choose the variable delay in a choice situation. As will be shown
below, these equations can also predict exactly what fixed delay should be equally preferred to the variable delay (i.e., an indifference rule).

Mazur (1984) obtained indifference points from pigeons using a wide variety of fixed and variable delays. Equations 1 and 2 were used to make predictions of their choices. The equations accounted for up to 95% of the variance at the group level, and fits were also very good with the individual subjects. Mazur (1986, 2007) also showed these equations could account for pigeons’ and rats’ choices between one and two (or three) delayed reinforcers.

Do humans also show a preference for variable over fixed delays to reinforcement? The results from previous studies with humans have been mixed. For example, using points in a video game as reinforcers, Kohn, Kohn, and Staddon (1992) found a preference for fixed over variable delays, a result that is the opposite of what has been found with nonhumans and the predictions of the hyperbolic equations. However, Locey, Pietras, and Hackenberg (2009) and Lagorio and Hackenberg (2010) found preferences for variability that were reasonably consistent with Equations 1 and 2 when the reinforcers were access to video clips. Lagorio and Hackenberg suggested that these mixed results might be due to the types of reinforcers used or other procedural differences.

In a recent set of experiments (McKerchar & Mazur, 2016), we asked people to make hypothetical choices between different amounts of money delivered after different delays. Some conditions used a variable-delay task, where the choices were between money delivered after fixed versus variable delays (e.g., $1000 in 1 year versus $1000 that had an equal chance of being delivered immediately or after 2 years), with the amount of the variable alternative adjusted across trials to identify indifference points. Other conditions used a double-reward task, which presented choices between one and two rewards delivered at different times (e.g., $1000
in 1 year versus $500 delivered immediately and $500 delivered in 2 years), with the amount of the double reward (Experiment 1) or the amount of the single reward (Experiment 3) adjusted across trials. All participants also completed a standard delay-discounting task, in which they chose between immediate and delayed rewards (e.g., $500 now versus $1000 in 1 year).

Although the indifference points in the standard discounting task were well described by Equation 1, there was no preference for variable delays or double rewards—a finding at odds with the predictions of Equation 2 and results obtained with nonhumans. To account for this, we proposed that in tasks where people answer a series of hypothetical questions about money, they are engaging a largely verbal repertoire. Thus, the behavior in these tasks is likely “rule governed,” rather than “contingency-shaped” (Skinner, 1963, 1984). Accordingly, the exact wording and structure of the questions may determine what rules people use in making their choices, and these rules may not be consistent with Equations 1 and 2. For instance, in our variable-delay tasks, because the fixed delay was exactly halfway between the two variable delays (e.g., a 1-year delay is halfway between no delay and a 2-year delay), participants may have decided that the fixed and variable options were roughly equivalent when the amounts associated with each alternative were equivalent (e.g., when both were $1000). Thus, the rule “these two alternatives are equal” could have led to our finding of no preference for variability.

Although the concept of rule-governed behavior might account for our previous results (McKerchar & Mazur, 2016), it still seems puzzling that the same participants whose indifference points were well described by a hyperbolic equation on the standard delay-discounting task showed no evidence of preference for variability on the variable-delay and double-reward tasks, as predicted by Equations 1 and 2. If the absence of a preference for variability in these experiments was due to some specific characteristics of the questions, using
different types of questions might produce different results. The present experiments were conducted to examine this possibility by using questions with a slightly different structure. Experiment 1 used a variable-delay task in which participants chose between an immediate reward and one with variable delays (e.g., $750 now versus $1000 immediately or after 1 year, with equal probability). Experiment 2 used a double-reward task in which participants chose between one immediate reward and two rewards, one of which was delayed (e.g., $750 now versus $500 now plus another $500 in 2 years). Experiment 3 used a slight variation of the double-reward task in which both rewards of the double-reward alternative were delayed (e.g., $750 now versus $500 in 1 month plus another $500 in 2 years). In all three experiments, the dollar amount for the immediate reward was adjusted over trials to find an indifference point. For instance, we might find that for one participant, $650 delivered immediately was about equally preferred to $500 now plus $500 in one year. Note that the alternative with the single-delay was always immediate; this is in contrast to McKerchar and Mazur (2016) in which the alternative with the single-delay was equal to the mean of the two delays of the variable-delay or double-reward alternative.

In all three experiments, participants also completed a standard delay-discounting task in which they chose between immediate and delayed rewards, and again the dollar amounts for the immediate reward were adjusted to estimate an indifference point. This allowed a point of comparison of the indifference points from the standard task with the indifference points obtained in the variable-delay and double-reward tasks. For any value of $K$, Equations 1 and 2 predict that indifference points should be smaller in the standard task than in the other two tasks. To illustrate this with an example, we can arbitrarily set $K = 0.05$. From Equation 1, we can calculate that for $1000 delivered in 2 years, $V = 1000/(1 + .05*24 \text{ months}) = 455$. Therefore,
$1000 in 2 years should be equivalent to $455 delivered immediately. Now consider the value of a variable option where $1000 might be delivered immediately or after 4 years, with $P_i = .5$ for each delay. From Equation 2, $V = .5 \times \frac{1000}{1.05^0 \text{ months}} + .5 \times \frac{1000}{1.05^{48 \text{ months}}} = $647. Even though the average delay to $1000 is 2 years in both of these examples, the value of the variable option is greater—$647 compared to $455. This is preference for variable delays.

Figure 1 shows the predictions of Equations 1 and 2 for a $1000 reward with a fixed or variable delay, with $K = 0.05$, for average delays of 0 to 5 years. This figure shows that except when delays are zero, the value of a reward with a variable delay should always be greater than a reward with a fixed delay. Accordingly, the predicted indifference points are greater in the variable task than in the standard task. This difference is predicted for any positive values of $K$ and $A$.

Besides serving as a point of comparison, the results from the standard delay-discounting task had a second important function. In each of the three experiments, a curve-fitting procedure was used to fit Equation 1 to the data from this task, and the best-fitting value of $K$ was found. This value of $K$ was then used in Equation 2 to make parameter-free predictions of the indifference points in the variable-delay and double-reward tasks—a method used in nonhuman studies on preference for variability (e.g., Mazur, 1984).

In each of the three experiments, three different amounts were tested: $100, $1000, and $5000. This was done both to test the generality of the results and to assess for magnitude effects. Previous studies have found that the rate of delay discounting decreases with larger dollar amounts, showing that people will choose to wait longer for a delayed reward when the amount of money is larger (e.g., Green, Myerson, & McFadden, 1997; Green, Myerson, & Ostaszewski, 1999; McKerchar et al., 2013).
In summary, the main questions addressed by these studies were: (1) Are there conditions under which humans will reliably show a preference for variable delays or double rewards when answering hypothetical questions about monetary rewards, and (2) If so, how accurately will the hyperbolic equations describe this effect?

**General Method**

**MTurk**

Across three experiments, 450 participants were recruited from Amazon Mechanical Turk (MTurk, https://www.mturk.com) to complete a Human Intelligence Task (HIT) hosted on SurveyMonkey (https://www.surveymonkey.com). The description of the HIT asked them to complete a survey to “help us better understand how people make decisions about money.” Participants were required to be at least 18 years old, reside in the U.S., and have completed at least 100 prior HITs with at least a 99% approval rating. All study procedures were approved, prior to their conduct, by the Institutional Review Board at the first author’s institution.

**Procedures**

All participants completed four sections of a web survey, in the following order: (a) informed consent, (b) a standard discounting task, (c) a variable-delay or double-reward discounting task, depending on the experiment, and (d) a demographic questionnaire. Table 1 provides demographic characteristics of the participants across the three experiments.

Three separate groups were formed in each of the three experiments (nine conditions total), which varied in the amount of the larger reward used in the standard and variable-delay (Experiment 1) or double-reward (Experiments 2 and 3) discounting tasks: $100, $1000, and $5000 (see Table 2). The amount was matched within each participant across both discounting
tasks (e.g., participants recruited for the $1000 condition from Experiment 1 completed a $1000
standard task and a $1000 variable-delay task).

Across the 450 participants of the nine conditions, the median duration for completing all
parts of the survey was 6.38 (interquartile range [IQR] = 4.68 – 8.82) minutes (individual
experiment times reported below). Each participant was compensated $0.50 for survey
completion. Payment was provided simply if a participant answered all survey questions; it was
never contingent on the pattern or quality of their responses. However, participants were
excluded from subsequent data analysis if their data were identified as nonsystematic.
Specifically, we excluded data from a participant if more than one of their indifference points
(beginning with the second) increased by 20% (or more) of the larger amount (e.g., $1000),
within either the standard or variable-delay task. This criterion accommodates a participant who
may have inadvertently selected the wrong alternative on one trial or who justifiably values
money at a single future delay greater than at an earlier delay (e.g., “I plan on moving in about 5
years, so I will wait to receive $5000;” see Rung, Argyle, Siri, & Madden, 2018). Depending on
the condition, between zero and four participants were excluded across the three experiments
(see below for more).

Experiment 1

Participants

Three separate groups of participants (N = 149) were recruited on separate days to complete a
standard delay-discounting and variable-delay discounting task, with each group corresponding
to a separate amount of the larger reward: $100 (n = 50), $1000 (n = 49), and $5000 (n = 50).

Procedure
Standard delay-discounting task. After providing informed consent, participants completed a standard discounting questionnaire asking them to make choices between smaller, immediate rewards (e.g., $500 now) and larger, delayed rewards (e.g., $1000 in 1 year). Participants were instructed, “Although the choices you make are not real, we ask that you please make your choices as though they are real. There are no right or wrong answers. We are simply interested in which you would choose, if you were to be offered these choices for real.” Following a practice question, participants began the task proper. The smaller, immediate reward was always listed first, and the larger, delayed reward was located directly below. Participants indicated their choice by clicking one of two radio buttons, located directly to the left of each alternative, followed by a “Next” button located below the two options. One question was presented on each screen.

On the first choice within each series, the amount of the smaller reward was always one-half the amount of the larger reward. For example, in the $1000 condition, the first question was:

Which would you rather have?

$500 now

$1000 in 1 month

If a participant chose the smaller reward, then the amount of this reward was decreased on the next screen by half its current amount (e.g., $250 now), while the amount and delay for the larger reward was fixed (e.g., $1000 in 1 month). Alternatively, if a participant chose the larger reward on the first trial, then the amount of the smaller reward increased on the next trial (e.g., $750 now). Thereafter, the amount of the smaller reward increased or decreased based on a participant’s choice for a total of five trials, with the size of each adjustment of the smaller
reward equal to one-half the size of the previous adjustment (see Du, Green, & Myerson, 2002). For example, the size of the five adjustments in the $1000 task, on the second through fifth trials, was: $250, $125, $62.50, and $31.25 (presented to nearest whole dollar).

The point of subjective equivalence between a smaller, immediate and a larger, delayed reward—the indifference point—was estimated as the amount of the smaller reward on the sixth choice-trial, had there been a sixth trial. Five series of trials were conducted in which the delay for the larger reward increased across each new series. The specific delays were: 1 month, 3 months, 1 year, 2 years, and 5 years.

**Variable-delay discounting task.** After completing the standard discounting task, participants received the following instruction in the variable-delay discounting task:

In this second survey, all questions ask you to choose between:

- A **guaranteed** amount of money delivered **immediately**, or
- A **guaranteed** amount of money, with a **50% chance** you will receive the money **immediately** and a **50% chance** you will receive the money **later**.

After answering a practice question, participants began the variable-delay task, with all trials consisting of a choice between a smaller, immediate and a larger, variable-delay reward. The smaller reward was always an amount of money to be received “now,” and the larger, variable-delay reward was an amount of money to be received either “now” or after a delay. For example, the first question in the $1000 condition was:

Which would you rather have?
$500 now

$1000

50% chance of receiving it now

50% chance of receiving it in 2 months

As in the standard task, the amount of the smaller reward on the first trial within a series was one-half the amount of the larger reward. If a participant chose the smaller (larger) reward, then the amount of the smaller reward increased (decreased) on the next screen. Within a series of trials, the two delays associated with the larger reward were constant. Across a series of trials, the “50% chance of receiving it now” was not changed, but the delay for the second “50% chance” increased as follows: 2 months, 6 months, 2 years, 4 years, and 10 years. These delays were chosen because the average of the two delays of the variable-delay alternative (e.g., average delay of “now” and 2 years) in each series matched the values of the delays of the larger, delayed alternative in the standard discounting task (e.g., 1 year); this allowed us to make a direct comparison of the indifference points between the two types of tasks. As in the standard task, an indifference point was estimated in each series as the amount of the smaller, immediate alternative on the sixth choice-trial, had there been a sixth trial.

Across the $100, $1000, and $5000 groups of Experiment 1, one, two, and three participants were excluded, respectively, using the criterion for nonsystematic discounting specified earlier (i.e., more than one indifference point exceeded the value of the preceding indifference point by 20% of the larger reward in either of the two discounting tasks). The median time (and IQR) to complete all parts of our survey in the $100, $1000, and $5000 groups was 5.37 (4.63 – 9.08), 6.60 (4.75 – 12.05), and 6.84 (5.03 – 8.67) minutes, respectively.
Results and Discussion

Figure 2 shows the group median indifference points obtained in the standard and variable-delay discounting tasks for the $100, $1000, and $5000 amounts. Table 3 shows the group- and individual-level estimates of $K$ from Equation 1 fitted to the standard task data, as well as the goodness-of-fit (root-mean-square error, $RMSE$) of Equations 1 and 2 in the standard and variable-delay tasks, respectively. $RMSE$ is the square root of the mean squared differences between the predicted and obtained values. As such, a smaller $RMSE$ indicates a better fit. We present $RMSE$ as a proportion of the larger amount so that fits can be meaningfully compared across amounts (e.g., $RMSE$ of 0.15 represents an average prediction error of 15%, regardless of the amount tested).

In the standard task, at all three amounts the group median indifference points were very well described by Equation 1 (solid lines), a simple hyperbola ($RMSE = .037$ to $.075$), as were the individual-level findings ($RMSE = .078$ to .085, see Table 3). Figure 3 shows the median (and IQR) of the individuals’ area under the curve (AUC) from the standard (left panel) and variable-delay (right panel) tasks at each of the three amounts. A one-way nonparametric ANOVA (Kruskal-Walis) indicated a main effect of amount in the standard task, ($\chi^2[2] = 9.79, p = .007$), with significantly greater discounting of $100 than $1000 and $5000; there was no difference in the AUC between $1000 and $5000 (Dunn’s post hoc, $ps$ adjusted for multiple comparison).

In the variable-delay task, the group indifference points tended to deviate from the predictions of Equation 2, particularly at the longer delays (Figure 2, dashed lines); this was reflected in the poorer fits obtained at each amount ($RMSE = .126$ to .160), relative to the group

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1 The AUC was calculated using both the conventional method (Myerson, Green, & Warusawitharana, 2001) and the ordinal method, recently described by Borges, Kuang, Milhorn, & Yi (2016). Both sets of findings yielded the same qualitative findings across all experiments; thus, we present only the conventional measure in all analyses.
fits of Equation 1 to the standard task data (see Table 3). At the individual level, we obtained $RMSE$s between .225 and .279 across the three amounts of the variable-delay task, values that were approximately three times larger than that obtained from the same individuals in the standard task with Equation 1. No main effect of amount was observed in this task ($p > .10$, Figure 3, right panel), as was observed in the standard task.

Recall that Equations 1 and 2 predict greater discounting of rewards with fixed delays than rewards with variable delays; thus, we should observe steeper discount functions (and smaller AUCs) in the standard task relative to the variable-delay task (Figure 1). Accordingly, we conducted two additional sets of analyses. First, we conducted a Wilcoxon signed-rank test (data not normally distributed) to statistically compare the AUCs obtained in the two types of tasks at each amount (e.g., AUCs from $100$ standard and $100$ variable-delay tasks). Second, we calculated the ratio of the AUC in the variable-delay task to the standard task at each amount, such that a value > 1.0 indicates a larger AUC (less discounting) in the variable-delay relative to the standard task (see Figure 4). We then compared this ratio score—the change in AUC ($\Delta AUC$)—to a theoretical value of 1.0 with Wilcoxon signed-rank tests at each amount.

At the small $100$ amount, the AUC in the standard task was significantly different from the AUC in the variable-delay task ($W = 635, p < .001$), with greater discounting in the standard task. However, at the two larger amounts ($1000$ and $5000$), the AUCs in the standard and variable-delay task did not differ significantly from each other. Figure 4 suggests the $\Delta AUC$ score only deviated from 1.0 at $100$, with less discounting in the variable-delay task at this amount; no apparent change occurred between the two types of task at $1000$ or $5000$. The Wilcoxon signed-rank tests supported this, indicating the $\Delta AUC$ was significantly different from 1.0 at $100$ ($W = 647, p < .001$), but not at $1000$ and $5000$ ($ps > .10$). Thus, at two of the three
amounts tested, a preference for variable delays was not observed—contrary to the predictions of Equation 2.

In summary, the data from this experiment were inconsistent overall with the predictions of Equation 2, and in two of the three conditions there was no evidence of preference for variable delays. However, choices of a different type might lead to a different outcome, and this possibility was examined in Experiment 2. As mentioned earlier Equations 1 and 2 also predict preference for multiple rewards over single rewards. As an example, imagine two choice alternatives: one consisting of a single $600 reward (delivered immediately) and another alternative consisting of two rewards, a $500 reward (delivered immediately) and a $500 reward (delivered in 2 years). Assuming $K = 0.05$, according to Equation 2 with $P_i = 1$, we can calculate the value of both rewards of the double-reward alternative, such that $V = 1(\frac{500}{1 + .05*0 \text{ months}}) + 1(\frac{500}{1 + .05*24 \text{ months}}) = $500 + $227. Thus, the values of the first and second components of the double-reward alternative are $500 and $227, respectively. Because $727 (\sum[$500 + $227]) is greater than the value of the single-reward alternative ($600), the participant should prefer the double-reward alternative. Furthermore, just as Equations 1 and 2 predict steeper discounting in a standard task relative to a variable-delay task (Figure 1), these equations also predict steeper discounting in the standard task relative to this double-reward scenario. This is preference for multiple rewards.

Mazur (1986, 2007) has shown that Equation 2 successfully predicts indifference points with rats and pigeons choosing between one and two (or three) reinforcers. Accordingly, the purpose of Experiment 2 was to evaluate the predictions of Equation 2 in predicting choice in a task where participants chose between one and two rewards (i.e., a double-reward choice procedure).
Experiment 2

Participants
As in Experiment 1, three groups of participants ($N = 151$) were recruited on separate days to complete a series of two discounting tasks, with each group receiving a different amount of the larger reward: $100 (n = 51)$, $1000 (n = 50)$, or $5000 (n = 50)$. However, rather than completing a standard delay-discounting and a variable-delay discounting task, all participants completed a standard discounting task followed by a double-reward discounting task.

Procedure

Standard delay-discounting task. The standard delay-discounting task of Experiment 2 was identical to that of Experiment 1.

Double-reward (immediate/long) discounting task. After completing the standard discounting task, participants received the following instruction in the double-reward discounting task:

In this second survey, all questions ask you to choose between:

An amount of money delivered immediately, or

Two amounts of money—one amount delivered immediately and another amount delivered at a later time.

After answering a practice question, participants began the double-reward task. All trials consisted of a choice between a single- and double-reward alternative. The single-reward alternative was always an amount of money to be received “now,” and the double-reward alternative was one amount of money to be received “now” and another amount after a delay (e.g., “in 2 years”). On the first trial within each series, the amount of the single-reward
alternative was one-half the total amount of the double-reward alternative. For example, in the $1000 condition, the first question was:

Which would you rather have?

$500 now

$500 now, plus $500 in 2 months

If a participant chose the single reward, then the amount of the single reward decreased by half its current amount on the next screen (e.g., “$250 now”). If instead a participant chose the double reward, then the amount of the single reward increased by half its current amount (e.g., “$750 now”). Thereafter, the amount of the single reward increased or decreased for a total of five trials, depending on a participant’s choice, with the size of each new adjustment equal to one-half the size of the previous adjustment.

Within each series of five choice trials, the double-reward alternative was constant. Across each new series, both amounts of the double-reward alternative (e.g., $500 and $500) and the first of the two delays remained unchanged (i.e., always “now”); however, the delay until the second of the two rewards increased (e.g., “in 4 years”). The precise delays of the second reward in the double-reward alternative were: 2 months, 6 months, 2 years, 4 years, and 10 years. As in Experiment 1, these delays were chosen because the average delay of the two rewards of the double-reward alternative (e.g., “now” and 2 years) matched the values of the delays used in the standard discounting task (e.g., 1 year); this allowed us to make direct comparisons of the indifference points in the standard and double-reward discounting task (recall that Equation 2 predicts shallower discounting in the double-reward task relative to the standard task). For ease
of communication, this double-reward task will be referred to as the “immediate/long” variant (cf. double-reward task of Experiment 3).

Four, two, and two participants were excluded from the $100, $1000, and $5000 conditions of Experiment 1, respectively, using the criterion for nonsystematic discounting specified previously. The median time (and IQR) to complete the entire survey in the $100, $1000, and $5000 groups was 5.80 (4.33 – 8.46), 5.91 (4.68 – 8.73), and 6.47 (4.54 – 8.92) minutes, respectively.

**Results and Discussion**

Figure 5 shows the group median indifference points obtained from participants in the standard and double-reward discounting tasks of Experiment 2. As in Experiment 1, Equation 1 provided an excellent description of choices in the standard task at all three amounts: $100, $1000, and $5000 ($RMSE = .033 to .057). Although Figure 6 (left panel) suggests a systematic increase in the AUC from the standard task as amount increases, this was not statistically significant (one-way Kruskal-Walis ANOVA, $p > .10$).

In the double-reward task, Equation 2 provided a good description of the group median indifference points, with an $RMSE$ of .037, .033, and .058 at the $100, $1000, and $5000 amounts, respectively. These group fits are similar to those obtained with Equation 1 in the standard task. At the individual level, the $RMSE$s from Equation 2 fit to the double-reward data were between .121 and .147, depending on the amount. Although these prediction errors are larger than those obtained in the standard task of this experiment, they are much smaller than those obtained from the variable-delay tasks of Experiment 1 (see Table 3). There was a suggestive, but nonsignificant effect of amount in this task (Figure 6, right panel; $\chi^2[2] = 4.77, p = .092$).
At each of the three amounts, the rate of discounting differed significantly between the standard and double-reward task, according to Wilcoxon signed-rank tests ($100: W = 733, $1000: W = 1064, $5000: W = 736; all ps < .001). The AUC score was significantly different from 1.0 at all three amounts ($100: W = 749, $1000: W = 1086, $5000: W = 746; all ps < .001), and Figure 7 shows it was greater than 1.0 at each of these amounts, suggesting a strong preference for multiple rewards.

For the reasons described earlier, Equation 2 predicts what the precise indifference points should be in a variable-delay and double-reward discounting procedure, and what follows from this is a predicted preference for variable-delays as well as a preference for multiple rewards. In contrast to the poor predictions of Equation 2 for Experiment 1, in the current experiment, a preference for multiple rewards was observed at each of the three amounts and this preference was in good quantitative agreement with the predictions of Equation 2. Although these latter findings were obtained when the double-reward alternative consisted of one immediate and one delayed reward, Equation 2 can make predictions for delays of any duration. To test the generality of the findings from Experiment 2, Experiment 3 used double rewards delivered after two delays, one short and one long. The main question was whether Equation 2 could also make accurate predictions when both components of the double-reward alternative were delayed (i.e., no immediate reward).

**Experiment 3**

**Participants**

We recruited three groups of 50 participants (N = 150) on separate days to complete both a standard and double-reward delay discounting task, with each group receiving a different amount of the larger reward, $100, $1000, or $5000. Within each group, the amount in the
standard and double reward task was identical; however, unlike Experiment 2, the double-reward alternative of this final experiment consisted of two non-zero delays, rather than one immediate and one delayed reward.

**Procedure**

**Standard delay-discounting task.** The standard delay-discounting task of Experiment 3 was identical to that of Experiments 1 and 2.

**Double-reward (short/long) discounting task.** After completing the standard discounting task, participants received the following instructions:

In this second survey, all questions ask you to choose between:

An amount of money delivered **immediately**, or

Two amounts of money--one amount delivered after a short delay and another amount delivered after a longer delay.

After answering a practice question, participants began the double-reward task of Experiment 3. All trials consisted of a choice between a single- and double-reward alternative. The single-reward alternative was always an amount of money to be received immediately and the double-reward alternative consisted of two rewards: one amount after a short delay and a second identical amount after a longer delay. For example, the first question in the $1000 condition was:

**Which would you rather have?**

$500 now

$500 in 1 week, plus $500 in 2 months
This double-reward task will be referred to as the “short/long” variant, to distinguish it from the double-reward procedure of Experiment 2, an “immediate/long” variant.

Across five trials within each series, the amount of the single reward was increased or decreased, depending on a participant’s choice, in a manner identical to the double-reward (immediate/long) task of Experiment 2. Across each new series of trials, the two amounts of the double-reward alternative were unchanged (e.g., $500 and $500), but the pair of delays associated with each were adjusted. The specific delays and order used across the series of choice trials were: 1 week and 2 months ($M = 1.13$ months), 1 week and 6 months ($M = 3.13$ months), 1 month and 2 years ($M = 12.50$ months), 2 months and 4 years ($M = 25.00$ months), and 5 months and 10 years ($M = 62.50$ months). These delays were chosen because: (a) the mean of the delays in each pair was similar (but not identical) to the delays in the standard discounting task and the preceding variable-delay and double-reward tasks, and (b) the time until the first delay, as a proportion of the overall delay until both rewards, was similar across the five pairs of delays.

Using the criterion for nonsystematic discounting specified previously, no participants were excluded from any of the three groups of Experiment 3. The median time (and IQR) to complete the survey in the $100, $1000, and $5000 groups was 6.24 (4.48 – 8.67), 6.78 (5.20 – 7.88), and 6.72 (4.93 – 8.51) minutes, respectively.

**Results and Discussion**

Figure 8 shows the group median indifference points obtained from participants in the standard and double-reward (short/long variant) discounting tasks of Experiment 3. As in the previous two experiments, Equation 1 provided an excellent description of choices in the standard task at all three amounts ($RMSE = .016$ to .069). Figure 9 shows the AUCs obtained
from the standard and double-reward (short/long) tasks of Experiment 3. A Kruskal-Wallis one-way ANOVA indicated a significant effect of amount in the standard task, ($\chi^2[2] = 11.97, p = .003$), with significantly greater discounting of $100 than $1000 and $5000; there was no difference in the AUC between $1000 and $5000 (Dunn’s post hoc, $p$s adjusted for multiple comparison).

In the double-reward (short/long) task, Equation 2 provided a good description of the group median indifference points at all three amounts, with an RMSE of .086, .055, and .047 at $100, $1000, and $5000, respectively. These prediction errors at the group level are comparable to those obtained in the double-reward (immediate/long) task of Experiment 2, and they represent about half the error obtained from the variable-delay task of Experiment 1 with Equation 2. At the individual level, the RMSEs from Equation 2 fit to the double-reward (short/long) data were between .130 and .172, depending on the amount. As in Experiment 2, the prediction errors at the individual level were larger than the group-level predictions (Table 3), but about half as large as the fits of Equation 2 fit from the individuals in the variable-delay task of Experiment 1. Although Figure 9 (right panel) suggests an amount effect in this double-reward task, this effect only approached significance ($\chi^2[2] = 4.89, p = .087$).

As in Experiment 2, the rate of discounting in this experiment differed significantly between the standard and double-reward task at each of the three amounts ($100: W = 929, p < .001; $1000: W = 643, p = .001; $5000: W = 511, p = .013$). The AUC was also significantly different from 1.0 at all three amounts ($100: W = 1051, p < .001; $1000: W = 695, p < .001$);

---

2 Because the delay until the first reward of the double-reward alternative was not fixed across each series of choices in the short/long task, the average delay until both rewards cannot be precisely determined for any subjective values that were not empirically obtained. Accordingly, the method of least squares could only be used here to predict the obtained indifference points, so we applied straight-line segments to connect them and approximate the predicted intermediate values.
$5000: W = 639, p = .002), and Figure 10 shows it was greater than 1.0 at all amounts. This suggest a strong preference for multiple rewards, even when both of the rewards are delayed.

**General Discussion**

In the standard delay-discounting task from all nine conditions of these three experiments, the participants’ indifference points followed a hyperbolic function that was well described by Equation 1. With $K$ treated as a free parameter in Equation 1, the $RMSE$ from these nine conditions was never larger than .075. These results are consistent with many previous studies that found hyperbolic discounting with human participants (e.g., Bickel et al., 1999; Johnson et al., 2015; Kaplan et al., 2014; Reynolds & Schiffbauer, 2004). In addition, there was evidence for a magnitude effect (e.g., Green et al., 1997; Green et al., 1999; McKerchar et al., 2013), in which the rate of delay discounting decreased as the amounts of money increased. Evidence for a magnitude effect was statistically significant in the standard task in Experiments 1 and 3, and there was a similar trend in Experiment 2, though it failed to reach statistical significance (see Figure 6). The performance of these participants recruited and tested using MTurk and SurveyMonkey was therefore very similar to what has been obtained in many previous experiments on human delay discounting.

The main purpose of these experiments was to determine whether participants would show a preference for the option with variable delays or two rewards in a way that is consistent with the predictions of Equation 2 (in the same way that preference for variable delays and multiple rewards in nonhumans has been shown to be consistent with this equation). The findings can be summarized quite simply: In the two experiments that used double-reward procedures—both the immediate/long variant used in Experiment 2 and the short/long variant used in Experiment 3—there was statistically significant evidence of a preference for the
alternative with multiple rewards, and the indifference points were fairly well predicted by Equation 2 at the group and individual level. In contrast, the indifference points obtained from the variable-delay task of Experiment 1 were not as well predicted by Equation 2, and except at the $100 amount, there was no evidence of a preference for variable delays. In Figure 11, the results from all nine conditions of Experiments 1 through 3 are compared to the predictions of Equation 2, and it can be seen that the predictions were quite accurate for Experiments 2 and 3 (middle and bottom rows) but less so for Experiment 1 (top row). It should be emphasized that these predictions were generated from Equation 2 with no free parameters, because the best-fitting value of $K$ from the corresponding standard task was used to make the predictions for the variable-delay and double-reward tasks.

To our knowledge, the results from Experiments 2 and 3 provide the first support of a preference for multiple rewards with human participants making choices with hypothetical monetary rewards, and it did so in the following two ways: (a) steeper discounting in the standard task relative to the double-reward tasks and (b) direct support for Equation 2 by predicting the obtained indifference points in the six double-reward conditions. However, in a few previous studies, humans displayed a preference for variable delays (consistent with Equation 2) when they actually experienced short delays that were followed by video clips as reinforcers (Lagorio & Hackenberg, 2010; Locey, Pietras, & Hackenberg, 2009). Kirby (2006) asked college students to choose between a small amount of money delivered immediately versus three amounts, each delivered after different delays. To make the task more realistic, participants actually received the money for one of their choices, selected at random. Using Equation 2 to estimate the value of the three delayed reinforcers, Kirby found the participants’ choices were consistent with the idea that the total value of the three-reinforcer package was
equal to the sum of the delay-discounted values of the three individual reinforcers. This finding is similar to the results from the double-reward conditions of the present experiments.

As we have already shown, Equation 2 predicts both preference for variability in the variable-delay task of Experiment 1 and preference for multiple rewards in the double-reward tasks of Experiments 2 and 3. Why then was there clear evidence of preference for multiple rewards in the double-reward tasks, but not a preference for variability in the variable-delay task? We suggest the distinction between contingency-shaped and rule-governed behavior we used to account for our previous findings on this topic may again be useful (McKerchar & Mazur, 2016). The behavior of nonhumans in experiments on preference for variable delays and of humans in those studies that used video clips as reinforcers can be called contingency-shaped (e.g., Locey, Pietras, & Hackenberg, 2009; Mazur, 1984), because they received actual and repeated experience with the delays and reinforcers. In contrast, we can describe the behavior of the participants in the present studies as rule-governed, because the questions were hypothetical and the participants were given no actual exposure to the delays and monetary rewards described in the questions. If this distinction between contingency-shaped and rule-governed behavior is accepted, then the question becomes: What sort of rules were the participants following in these different tasks?

For the standard and double-reward tasks, not much needs to be said because the choices of the participants were consistent with hyperbolic discounting functions described by Equations 1 and 2. We are not suggesting that participants were performing calculations using these equations (any more than nonhumans do when their choices conform to these equations), but simply that their choices were consistent with them. But some other type of rule-governed behavior must have been at work in the variable-delay conditions, because the results were
distinctly different. One notable difference is that in the double-reward task, the delays and amounts of money are certain, whereas in the variable-delay task, the delay is uncertain and possibly a long one. If the variable-delay option is chosen, there is a 50% chance of receiving nothing now and having to wait a long time for the money. It could be that this uncertainty of delay, particularly with relatively long delays, is what decreased the subjective value of the variable-delay option more than that predicted by Equation 2. As shown in Figure 11 (top row), the departures from the predictions were greatest for the choices that involved the longest delays.

Our previous research on this topic included some tasks with a double-reward alternative, but in these tasks participants did not show a preference for multiple rewards in a way that was predicted by Equation 2 (McKerchar & Mazur, 2016). This difference from the present results of the double-reward tasks may be due to a different structure of the questions, or what Kahneman and Tversky (1979) called the “framing” of the questions. The questions in the previous study all included a choice between a double reward (e.g., $500 delivered immediately and $500 delivered in 2 years) and a single reward with a delay that was the average of the two delays for the double-reward option ($1000 in 1 year). This type of question may have served to emphasize that the average delay for the double reward was equal to the delay for the single reward, and it might have prompted participants to treat the two options as roughly equal in value. In contrast, in the present experiment the choices were between a double reward (e.g., $500 delivered immediately and $500 delivered in 2 years) and an immediate reward (e.g., $600 now). This question structure may have prompted participants to evaluate the present value of the double-reward option, taking into account the total values of the two rewards and their respective delays. The result was that participants’ choices in the present experiment were well predicted by Equation 2, whereas those of McKerchar and Mazur (2016) were not. This difference emphasizes the point
that the exact structure and wording of the questions can have an important effect on decision making when participants make hypothetical choices about money (DeHart, Friedel, Frye, Galizio, & Odum, 2018; DeHart & Odum, 2015; Klapproth, 2012; LeBoeuf, 2006; Read, Frederick, Orsel, & Rahman, 2005).

Unlike in the standard discounting task, no significant magnitude effect was found in the variable-delay and double-reward tasks. It is not clear why there should be a magnitude effect in the standard task but not in the other tasks. However, as shown in the right panels of Figures 3, 6, and 9, there was a small tendency for the AUC to increase as amount increased in all three experiments.

In summary, we found that when human participants made hypothetical choices involving either single delayed reinforcers (as on the standard task) or two reinforcers delivered after different delays (as on the double-reward task), their choices were well described by a hyperbolic decay equation. These findings are consistent with what has been found with animal subjects, and with a few studies in which humans received actual delayed reinforcers. However, when the variable alternative consisted of an option that might be delivered either immediately or after a long delay, the subjective value of this option was lower than predicted by the hyperbolic decay equations, and no consistent preference for variability was found. Although the predictions of Equation 2 are the same for the double-reward and variable-delay tasks, the participants made distinctly different choices in these two cases. It seems likely that the uncertainty of the delays in the variable-delay task was responsible for the difference in results.
References


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Table 1

Demographics of participants from Experiments 1 through 3.

<table>
<thead>
<tr>
<th>Demographic</th>
<th>Experiment 1</th>
<th>Experiment 2</th>
<th>Experiment 3</th>
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<tbody>
<tr>
<td></td>
<td>$100</td>
<td>$1000</td>
<td>$5000</td>
</tr>
<tr>
<td>n</td>
<td>50</td>
<td>49</td>
<td>50</td>
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<tr>
<td>Age in years, mean (SD)</td>
<td>32.8</td>
<td>36.4</td>
<td>36.9</td>
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<tr>
<td>Sex (% female)</td>
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<td>49</td>
<td>50</td>
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<tr>
<td>Ethnicity (% White/Caucasian)</td>
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<td>67</td>
<td>76</td>
</tr>
<tr>
<td>Marital status (%)</td>
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<td>50</td>
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<td>6</td>
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<td>Separated</td>
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<td>0</td>
</tr>
<tr>
<td>Widowed</td>
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<td>0</td>
<td>2</td>
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<tr>
<td>Annual Income (%)</td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>4</td>
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<td>14</td>
<td>6</td>
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<td>0</td>
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<td>$175,000 to $199,999</td>
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<td>2</td>
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<tr>
<td>Education (%)</td>
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<td>0</td>
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<tr>
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<td>13</td>
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## Variable Delays and Double Rewards

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<th>31</th>
<th>25</th>
<th>20</th>
<th>22</th>
<th>32</th>
<th>20</th>
<th>16</th>
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<td>Some college, no degree</td>
<td>18</td>
<td>8</td>
<td>17</td>
<td>18</td>
<td>20</td>
<td>12</td>
<td>10</td>
<td>10</td>
<td>22</td>
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<tr>
<td>Associate’s degree</td>
<td>37</td>
<td>53</td>
<td>29</td>
<td>39</td>
<td>42</td>
<td>50</td>
<td>44</td>
<td>41</td>
<td>26</td>
</tr>
<tr>
<td>Bachelor’s degree</td>
<td>8</td>
<td>10</td>
<td>10</td>
<td>4</td>
<td>12</td>
<td>10</td>
<td>0</td>
<td>10</td>
<td>18</td>
</tr>
</tbody>
</table>

*Note.* Due to rounding, numbers do not always sum to 100%.
Table 2

*General format and specific examples of the discounting tasks from Experiments 1 through 3.*

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Task</th>
<th>Which would you rather have? Choose one.</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 3</td>
<td>Standard</td>
<td>$X$ now</td>
<td>$500$ now</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A$ after delay of $D$</td>
<td>$1000$ in $1$ month</td>
</tr>
<tr>
<td>1</td>
<td>Variable Delay</td>
<td>$X$ now</td>
<td>$500$ now</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A$ 50% chance of receiving it now 50% chance of receiving it after delay of $2D$</td>
<td>$1000$ 50% chance of receiving it now 50% chance of receiving it in $2$ months</td>
</tr>
<tr>
<td>2</td>
<td>Double Reward (immediate/long)</td>
<td>$X$ now</td>
<td>$500$ now</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Half of $A$ now, plus half of $A$ after delay of $2D$</td>
<td>$500$ now, plus $500$ in $2$ months</td>
</tr>
<tr>
<td>3</td>
<td>Double Reward (short/long)</td>
<td>$X$ now</td>
<td>$500$ now</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Half of $A$ after a short delay, plus half of $A$ after a delay of $2D$</td>
<td>$500$ in $1$ week, plus $500$ in $2$ months</td>
</tr>
</tbody>
</table>

$X$ is the amount of the first (immediate) alternative, which increased or decreased across five trials for each value of $D$, based on the participant’s choices. $A$ is the amount of the second alternative, which was $100$, $1000$, or $5000$, depending on group assignment. In different sets of questions, $D$ was $1$ month, $3$ months, $1$ year, $2$ years, and $5$ years. See text for more details about the specific questions.
Table 3

*Estimates of K and model fits (RMSE) with the simple hyperbola (Equation 1) in the standard delay-discounting task and RMSE from the general hyperbola (Equation 2) in the variable delay (Experiment 1) and double-reward (Experiments 2 and 3) tasks of Experiments 1 through 3. Values were determined from fits to the group medians and the median of the individuals.*

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Task</th>
<th>Group Median</th>
<th>Median of Individuals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>K</td>
<td>RMSE</td>
</tr>
<tr>
<td>1</td>
<td>Standard $100</td>
<td>0.235</td>
<td>.066</td>
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<tr>
<td></td>
<td>Variable Delay $100</td>
<td>.153</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Standard $1000</td>
<td>0.050</td>
<td>.075</td>
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<tr>
<td></td>
<td>Variable Delay $1000</td>
<td>.126</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Standard $5000</td>
<td>0.043</td>
<td>.037</td>
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<tr>
<td></td>
<td>Variable Delay $5000</td>
<td>.160</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Standard $100 (immediate/long) $100</td>
<td>0.075</td>
<td>.057</td>
</tr>
<tr>
<td></td>
<td>Standard $1000</td>
<td>0.053</td>
<td>.036</td>
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<tr>
<td></td>
<td>Double Reward (immediate/long) $1000</td>
<td>.033</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Standard $5000</td>
<td>0.042</td>
<td>.033</td>
</tr>
<tr>
<td></td>
<td>Double Reward (immediate/long) $5000</td>
<td>.058</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Standard $100 (short/long) $100</td>
<td>0.301</td>
<td>.055</td>
</tr>
<tr>
<td></td>
<td>$100</td>
<td>.086</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Standard $1000</td>
<td>0.057</td>
<td>.069</td>
</tr>
<tr>
<td></td>
<td>Double Reward (short/long) $1000</td>
<td>.055</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Standard $5000</td>
<td>0.052</td>
<td>.016</td>
</tr>
<tr>
<td></td>
<td>Double Reward (short/long) $5000</td>
<td>.047</td>
<td></td>
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</table>

*Note. K values obtained from the standard discounting task with Equation 1 for each group were applied to Equation 2 to generate the indifference predictions in the variable-delay and double-reward tasks for the same group of participants (e.g., K value of Standard $100 group from Experiment 1 applied to Equation 2 to predict the same group’s discounting in the variable-delay $100 task). Thus, the hyperbola*
predictions of Equation 2 in the variable-delay and double-reward tasks had no free parameters. See text for more.
Figure 1. The predicted indifference points for a $1000 reward ($K = 0.05$) with either a fixed (solid line) or variable (dashed line) delay, estimated with Equations 1 and 2, respectively. Note that at all delays (except zero), the predicted value of a reward with a variable delay is greater than the value of a reward with a fixed delay.
Figure 2. Group median indifference points and the fits of Equations 1 (solid lines) and 2 (dashed lines) obtained from the standard (closed circles) and variable-delay (open square) discounting tasks in the three groups of Experiment 1. The left, middle, and right column of panels correspond to participants who received the $100, $1000, or $5000 amounts, respectively. RMSE is the mean prediction error in each task at each amount. See text for more.
Figure 3. Median of individual participants’ AUCs (and IQR) at the $100, $1000, and $5000 amounts in the standard (left panel) and variable-delay (right panel) discounting tasks of Experiment 1.
Figure 4. Change in area under the curve (Δ AUC, geometric mean [95% CI]) between the variable-delay and standard discounting tasks of Experiment 1 at the $100, $1000, and $5000 amounts. Δ AUC of 1.0 = no change, Δ AUC > 1.0 = less discounting (larger AUC) in variable-delay relative to standard discounting task. * = Significantly different from 1.0.
Figure 5. Group median indifference points and the fits of Equations 1 (solid lines) and 2 (dashed lines) obtained from the standard (closed circles) and double-reward (open square) discounting tasks in the three groups of Experiment 2. The left, middle, and right column of panels correspond to participants who received the $100, $1000, or $5000 amounts, respectively. RMSE is the mean prediction error in each task at each amount. See text for more.
Figure 6. Median of individual participants’ AUCs (and IQR) at the $100, $1000, and $5000 amounts in the standard (left panel) and double-reward (right panel) discounting tasks of Experiment 2.
Figure 7. Change in area under the curve (Δ AUC, geometric mean [95% CI]) between the double-reward and standard discounting tasks of Experiment 2 at the $100, $1000, and $5000 amounts. Δ AUC of 1.0 = no change, Δ AUC > 1.0 = less discounting (larger AUC) in double-reward relative to standard discounting task. * = Significantly different from 1.0.
Figure 8. Group median indifference points and the fits of Equations 1 (solid lines) and 2 (dashed lines) obtained from the standard (closed circles) and double-reward (short/long variant, open squares) discounting tasks in the three groups ($100, $1000, and $5000) of Experiment 3. The least-squares method was used only to make point predictions in this task, and straight-line segments were used to connect these predictions (see Footnote 2). Note: unlike in Experiments 1 and 2, the average delays in the two type of tasks are not matched identically. See text for more.
Figure 9. Median of individual participants’ AUCs (and IQR) at the $100, $1000, and $5000 amounts in the standard (left panel) and double-reward (right panel) discounting tasks of Experiment 3.
Figure 10. Change in area under the curve (Δ AUC, geometric mean [95% CI]) between the double-reward (short/long variant) and standard discounting tasks of Experiment 3 at the $100, $1000, and $5000 amounts. Δ AUC of 1.0 = no change, Δ AUC > 1.0 = less discounting (larger AUC) in variable-delay relative to standard discounting task. * = Significantly different from 1.0.
Figure 11. Indifference points and the predictions of Equation 2 fit to the variable-delay (top row), double-reward (immediate/long variant, middle row), and double-reward (short/long variant, bottom row) tasks of Experiments 1, 2, and 3, respectively. Note: These fits had no free parameters, as $K$ from Equation 1 fit to the data from the standard task was used as the $K$ in Equation 2. See text for more.